With regards to storage and accuracy, the method described in this Note used 1194 bytes of storage on the 360/65, whereas the method of Heffron and Watson required 1324 bytes, or over 10% more storage. In addition Table 1 shows that, except for the altitude range of from 0.2×10^9 ft to 0.2×10^{16} ft, the latitude error is virtually identical and, for all practical purposes, the altitude error is the same throughout. Both methods, therefore, have an over-all accuracy of 15 out of 16 digits except where noted.

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Cryogenic Know-How as Applied to Inground LNG Storage

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THE concept of inground storage developed from the need to find an economical means of storing large volumes (i.e., more than 400,000 barrels) of liquefied natural gas (LNG). The costs of storing such volumes in conventional double-walled, insulated metal tanks are high. Moreover, the large-scale conventional storage tanks have not yet been designed and built; consequently, there is no assurance that LNG can be stored this way in an economic manner.

Using the ground itself as a container for LNG, or any cryogen, is, in theory at least, a simple means of overcoming the problems encountered with conventional tank storage. One has merely to excavate a large, cylindrical hole in the ground, cover it with an insulated roof, and fill the hole with LNG. The earth walls surrounding the hole cool, freezing any water present in the formation, and behave, ideally, as an infinite-slab heat sink. Although heat transfer is very high initially, it eventually decreases and boil-off reaches an acceptable level.

While the technique of inground storage is theoretically simple, the reduction to practice is not. A typical inground storage system—the CAMEL plant in Arzew, Algeria—has reported boiloff losses of about 0.3% per day, which is considered acceptable for this "base load" (shipment depot type) plant. However, two similar systems in the United States, one at Hackensack, N.J., and one at Hopkinton, Mass., have been abandoned because of high boil-off losses. Another facility built for the British Gas Council at Canvey Island, Great Britain, has recently gone into full operation and LNG is being fed into the inground tanks. A stabilized boiloff value has not been determined, but indications are that boiloff will be considerably higher than had been predicted.

This Note describes some of the factors that affect the boiloff rate of inground storage systems.

Prefreeze and Excavation

To excavate successfully a large, deep hole in the earth, one must make the containing walls strong enough to eliminate cave-ins or slides. In addition, since typical inground tanks are 100–200 ft deep, some provision must be made to control water flow into the excavation.

Saturated frozen earth is both a water barrier and a rather strong material; thus, one way to excavate an inground storage tank is to insert a number of freeze pipes just outside the perimeter of the planned excavation (see Fig. 1). Refrigerant is pumped through these pipes at a temperature of −25 to −100°F, and the prefreeze system must operate continuously during the entire excavation period. After an annular wall of ice and frozen earth is formed, the hole is excavated. Once LNG filling starts, the prefreezing operation can be discontinued.

Since excavation often utilizes controlled blasting, the prefreeze pipes must be constructed of a metal which is not susceptible to brittle fracture at prefreeze temperatures. The mechanical properties of the frozen soil at reduced temperatures are extremely important also, since the tank roof usually is supported by the frozen soil. Creep rates and fracture characteristics of the soil must be known, and allowances must be made for gradual creep of the loaded ice-soil medium.

Thermal Performance of Inground Tanks

The most important heat leak to the tank contents occurs from the surrounding earth through the side walls and bottom. However, radiation from the roof can also be significant unless it is insulated well enough to isolate the tank from fluctuations in ambient conditions.

Mathematical models have been used to predict the performance of existing and proposed inground storage units. Hashemi and Sliepcevich,[1] for example, have developed a computer model which can be used to estimate cooldown and long-term boiloff performance for such tanks, provided the thermal properties of the system are known and no extraneous heat sources are present.

Thermal Properties

In general, soils and rocks are anisotropic; consequently, there is considerable variation in heat capacity, thermal conductivity, and density even for adjacent samples. In order to reasonably explain these variations in the properties of interest, we calculated how quartz content and orientation with respect to direction of heat flow affect thermal conductivity in a base rock.

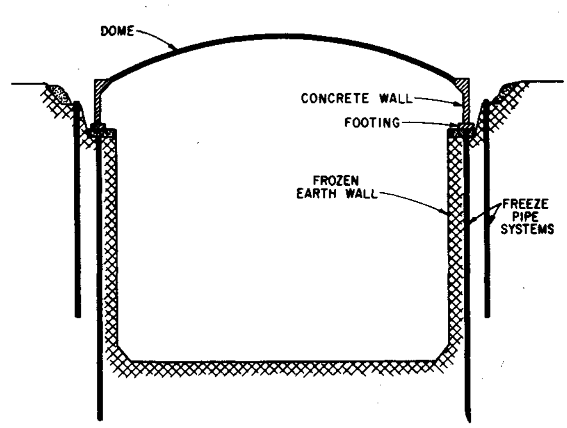

Fig. 1 Cross section of inground LNG storage tank.

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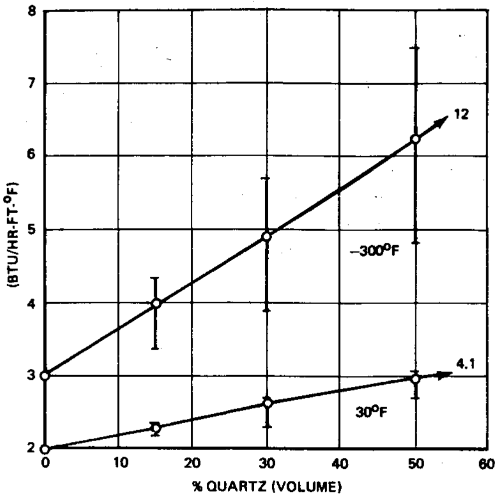

Fig. 2 Effect of quartz content on thermal conductivities of rock.

A noncrystalline base rock (i.e., no quartz content) has a thermal conductivity of close to 2 Btu/hr-ft-°F at 30°F and 3 Btu/hr-ft-°F at −300°F. In the direction of the six-fold axis of symmetry, pure quartz has a thermal conductivity of 4.1 Btu/hr-ft-°F at 30°F and 12 Btu/hr-ft-°F at −300°F. If values of thermal conductivity are calculated for a given percentage of quartz in base rock, consideration must be given to the orientation of the base rock and quartz layers with respect to heat flow. Three models were considered: 1) heat flow through parallel layers of rock and quartz, 2) heat flow through rock and quartz in series, and 3) heat flow through a cubic array of equal sized cubes of quartz dispersed uniformly through the base rock.

Series orientation of layers gave the highest values of k, parallel layers the lowest values of k, and the uniformly dispersed cubic array an intermediate value. A curve, as shown in Fig. 2, was drawn through the intermediate values, which we felt were reasonable to use for heat flow calculations.

Water Migration

Exclusive of underground rivers or major streams, the normal flow rates of water migrating in various subsurface soils appear to have a negligible effect on the heat leak into the contents of an inground storage tank.

Cracking of Subsurface Formation

During excavation, the prefreeze system maintains the material adjacent to the tank-wall faces in the −25 to −100°F range. When LNG (−259°F) is introduced into the

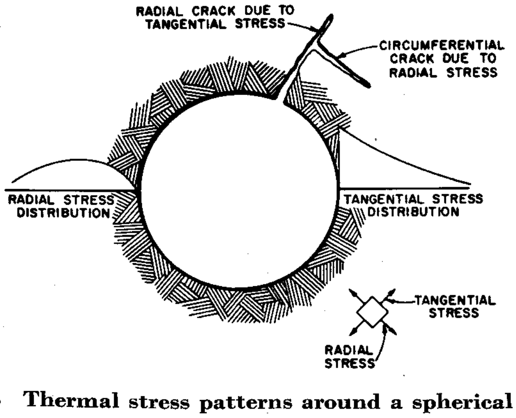

Fig. 3 Thermal stress patterns around a spherical hole.

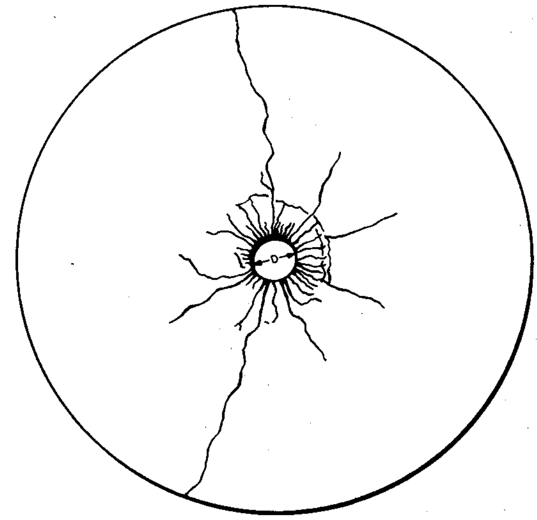

Fig. 4 Typical fracture pattern for internal loading of a circular disc with centerhole.

cavity, severe thermal stresses may develop in the surrounding medium. These stresses can be estimated by considering the storage tank as either a spherical or cylindrical cavity. Analyses[2] indicate that the thermal stress distributions for the sphere and for the cylinder are very similar. The magnitude and distribution of the stress fields surrounding a spherical cavity are shown in Fig. 3, with two principal stress patterns; a tangential or hoop stress field, and a radial stress field.

Using typical values for the coefficient of thermal contraction and the modulus of elasticity of rock, tangential stress levels at the liquid surface of a large, rock, inground LNG tank are in the range of 5000–10,000 psi, while maximum radial stress values are about half as large. Both tangential and radial stresses are tensile, and since most rocks and frozen soils are brittle-like materials, it can be assumed that the entire response of the structure to the thermal loading would consist of strain, or cracking, rather than stress.

The effect of the bottom of the excavation is difficult to analyze, but certainly, the stresses near the bottom corners of the excavation will be influenced by the discontinuity and the degree of cracking will be more severe at this location.

Assuming that the tensile strength of the frozen material is zero, the total width of the resulting cracks or contraction in the wall of the tank may be calculated. The value for the circumferential contraction may be interpreted as the sum of the widths, or openings, of all the radial cracks produced by the tangential stress field.[2] This approach leads to a total strain of 30 in. for a 200-ft-diam cylinder, based on a coefficient of thermal contraction of 1×10^{-5} in./in. °F (typical for rock), a temperature differential of 300°F and a Poissons ratio of 0.2.

The extent to which these cracks penetrate the surrounding medium depends on the actual tensile strength of the frozen rock and the flow of LNG into the cracks.

When loadings are imposed on the walls of a circular or spherical cavity, they cause crack patterns in the surrounding medium. These cracks have been shown[3–5] to affect a region whose dimensions are of the same order as the crack length. For this configuration to hold true, the density of the cracks must decrease with distance from the loaded surface, as illustrated in Fig. 4. Near the surface of the hole, many short cracks develop, but because they are short, they cannot relieve the stresses over a large region. A few of these cracks continue to greater depths, and ultimately, two cracks are sufficient to relieve the stresses to the required degree over the entire disk. Note also that circumferential cracks develop, which is consistent with the distribution of radial stress fields predicted from theoretical study. Fracture patterns similar to those shown in Fig. 4 have been observed from thermal loadings during experimental studies conducted at ADL.[6]

Applying this experimental evidence to an LNG storage tank excavated in rock, we can conclude that a minimum of perhaps six major cracks propagating to a distance of about one radius from the surface of the tank would be needed to relieve the stress field; if the crack propagation extends to only a $\frac{1}{2}$ radius, approximately twenty cracks could be expected.

This hypothesis assumes that the medium is essentially homogeneous and isotropic, and that the LNG does not penetrate into the larger cracks to any significant degree, which could result in further crack propagation. Actually, the fracture pattern will be altered, perhaps markedly, by LNG flow as well as by the specific characteristics of the rock strata, such as the existing crack and fault systems, schistosity, dip of the strata, anisotropy, etc.

Thermal Effects due to Cracking

Since it appears likely that large cracks will develop in frozen-rock, inground tank structures, the effect of such cracking on boiloff losses is of interest. These effects will tend to increase the predicted heat loads because the wetted surface area will be greater than anticipated.

Cracks occurring in or near the tank bottom, will be downward-directed and can produce a geysering[7] action that will introduce a very large heat leak into the tank.

If cracking occurs, we would expect boiloff to be at least 50% greater than that predicted for the uncracked case. The economics of peak shaving plants are very sensitive to boiloff losses. Therefore, cracking problems make rock inground storage less attractive than above-ground storage tanks.

Conclusion

The thermal performance of rock inground LNG storage tanks is adversely affected because major cracking occurs in the frozen rock and soil structure around the excavation. These cracks result in boiloff losses that we estimate to be at least 50% greater than would be predicted for an uncracked configuration. The cost savings possible with large-scale inground storage can be realized only if cracking is minimal or if an impermeable, inexpensive liner or sealer can be developed to prevent LNG flow into cracks. Techniques to maintain structural integrity during construction and operation must also be evaluated in the light of crack formation.

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Efficiency of a Pressurization Process

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ASTRONAUTS carry their air or oxygen supply with them in small pressure vessels that will have to be refilled periodically from larger containers of compressed gas. Filling of small pressure vessels from large supply containers also has more mundane applications (e.g., the charging of a shock-tube driver section).

The inefficiency of the procedure of charging vessels from larger supply containers is due to the fact that the gas in the supply vessel becomes unavailable when its pressure falls below the minimum required pressure of the vessel being charged. By using a second supply container to top off the vessel being charged, more of the gas in the first supply vessel becomes available. If several supply containers are used sequentially, it is clear that if the first supply container were nearly empty when the last fell below the required pressure, the charging procedure would make efficient use of the gas in each of the supply containers.

Although this type of charging procedure has been used for many years, it appears that no analysis of it has been published. This Note presents an analysis for an array of an arbitrary number of supply vessels, with equal volume and equal initial pressure, and an arbitrary number of chargings of smaller vessels, all with the same volume and initial pressure. The charging is done slowly, so conditions may be assumed to be isothermal. This analysis shows the effects of various parameters on the efficiency of gas utilization. Certain other useful results also are obtained.

Consider the pressurization of a number of vessels all with volume V_0 initially at pressure P_0 sequentially from an arbitrary number of supply vessels all with volume V^0 initially at a pressure P^0. For an isothermal expansion of a perfect gas the pressure of a supply vessel (and the pressure in the receiver vessel which it is charging since they are equal) is related to the supply vessel pressure and the receiver vessel pressure before these two pressures were equalized. Equating the mass lost by the supply container to that gained by the receiver vessel and employing the equation of state for a perfect gas the expression for the pressure of the supply and receiver vessels is obtained as

$$V^0(P_m{}^{n-1} - P_m{}^n) = V_0(P_m{}^n - P_{m-1}{}^n) \quad (1)$$

where

$P_m{}^{n-1}$ = pressure of supply vessel m after it has been used $n - 1$ times (initial pressure of m for the nth use)
$P_{m-1}{}^n$ = pressure of supply vessel $m - 1$ after it has been used n times (initial pressure of the vessel being charged by m on its nth use)
$P_m{}^n$ = the pressure of supply vessel m after its nth use and the pressure of the nth vessel that has been charged by m.

The preceding difference equation may be put into the standard form

$$P_m{}^n - \alpha P_m{}^{n-1} = (1 - \alpha)P_{m-1}{}^n \quad (2)$$

where

$$\alpha = V^0/(V^0 + V_0) = \text{volume ratio} < 1$$

For $m = 1$, the case of a single supply vessel, and since $P_0{}^n$ is

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